Homework

1. In △ABC, AD is the angle bisector of ∠A. M is the midpoint of BC. ME // AD intersects AC at F and intersects the extension of BA at E. Prove that BE = CF = (AB + AC)/2.

2. In pentagon ABCDE, ∠ABC = ∠AED = 90°, ∠BAC = ∠EAD, and M is the midpoint of CD. Prove that MB = ME.

3. In trapezoid ABCD with non-parallel sides AB = CD, rotate △ABC about point C by some angle to obtain △A'B'C'. Prove that the midpoints of A'D, BC, and B'C' are collinear.

4. On the exterior of sides AB and AC of △ABC, construct right triangles ABD and ACE such that ∠ABD = ∠ACE. M is the midpoint of BC. Prove that DM = EM.

5. Given convex quadrilateral ABCD with an interior point O such that ∠AOB = ∠COD = 120°, AO = OB, and CO = OD. Let K, L, M be the midpoints of AB, BC, CD respectively. Prove: (1) KL = LM; (2) △KLM is an equilateral triangle.

6. △DEF is obtained by translating △ABC along side BC, and DE passes through the midpoint M of AC. Prove that M is the midpoint of DE.

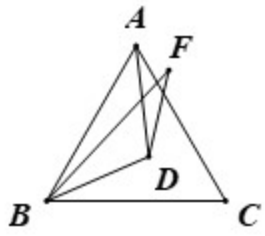
7. In isosceles △ABC with vertex angle ∠BAC = 100°, extend AB to D such that AD = BC. Find the measure of ∠BCD.

8. In isosceles right △ABC with ∠C = 90°, AM is the median to side BC, and CN ⊥ AM intersects AB at N. Prove that ∠NMB = ∠CMA.

9. On the hypotenuse AB of isosceles right △ABC, take points M and N such that ∠MCN = 45°. Let AM = m, MN = l, BN = n. Construct a triangle with side lengths l, m, n. What special property does this triangle have?

10. In convex pentagon ABCDE, ∠ABC = ∠AED = 90°, ∠DAC = 30°, ∠BAE = 70°, F is the midpoint of CD, and FB = FE. Find the measure of ∠BAC.

11. As shown in the figure, D is a point inside equilateral △ABC such that DB = DA, BF = AB, and ∠DBF = ∠DBC. Find the measure of ∠BFD.



12. In △ABC, AC = BC, ∠ACB = 90°, D is a point on AC, and AE is perpendicular to the extension of BD at E, with AE = BD. Prove that BD is the angle bisector of ∠ABC.

13. In △ABC with ∠C = 90°, D is a point on AB. Construct DE ⊥ BC at E such that BE = AC, BD = √2, and DE + BC = 1. Prove that ∠ABC = 30°.

14. In acute △ABC, the altitudes AA_1 and CC_1 intersect at point O. Prove that if OA = OC, then △ABC is isosceles.

Before you've exhausted all your efforts in exploring and thinking, it's best not to refer to the hints and answers in the assignment
https://archive.org/details/mastery-level-4-geo-2-congruent-triangles-homework-a-h

There are many individual problems and question banks available on internet, some of which match the content of this lecture, so they can be found and practiced too, but the overall efficiency is low.

If you are interested in increasing your practice set on this theme, can request additional exercises from xuyouzhi2015@gmail.com.